Int. J. Multiphase Flow Vol. 7, pp. 41–46
Pergamon/Elsevier, 1981. Printed in Great Britain

0301-9322/81/0201-0041/$02.00/0

EXPERIMENTAL WALL CORRECTION FACTORS OF SINGLE SOLID SPHERES IN TRIANGULAR AND SQUARE CYLINDERS, AND PARALLEL PLATES

A. Miyamura,† S. Iwasaki‡ and T. Ishii§
Department of Chemical Engineering, Yokohama National University, Yokama 240, Japan

(*Received* 28 *August* 1979; *in revised form* 27 *June* 1980)

Abstract—To examine wall effects, the steady state settling velocities of single solid spheres of various diameters were measured in triangular and square cylinders, and between parallel plates, which were filled up with highly viscous aqueous milet jelley solutions. The wall effect for the steady state motion of single spheres in a narrow space has to date been correlated by the wall correction factor. The wall correction factors of single solid spheres were correlated in such apparatus in the creeping flow region.

INTRODUCTION

Particles interact with other particles in hindered settling and particulate fluidization. The interaction among the particles was correlated by the concept of the void function proposed by Ito (1957, 1962). The previous theoretical and experimental void functions were summarized by Ishii (1965). On the other hand, particles moving in a narrow space are influenced by the wall. The wall effect has been correlated by the concept of the wall correction factor.

There have been many theoretical and experimental studies of wall correction factors of single solid spheres in circular cylinders in the creeping flow region, which have been summarized by Iwaoka & Ishii (1979). Recently, Iwaoka & Ishii (1979) reported the wall correction factors of single solid spheres in circular cylinders in the creeping flow region and their data agreed very well with the exact solutions of Haberman (1956), and Paine & Scherr (1975).

However, there have been few experimental and theoretical studies on the wall correction factors of single solid spheres in various apparatus other than circular cylinders.

In this study, the wall correction factors of single solid spheres were measured in triangular and square cylinders, between parallel plates in the creeping flow region.

The purpose of this series of studies is to measure the wall effect of single solid spheres under steady state motion in various apparatus in the creeping flow region. These studies also resulted in development of new falling ball viscometers.

2. EXPERIMENTAL

The experimental apparatus used were made from transparent acryl resin plates and are shown in figures 1–3. The triangular cylinder with the triangular jacket was 100 cm in height and 5 mm in perpendicular distance from the center to each inner side of a regular triangular cross-section of the cylinder. The square cylinder with the square jacket was 100 cm in height and 1 cm in an inner side length. The parallel plates with the rectangular jacket were 100 cm in height, 0.99 cm in shorter width and 10 cm in longer width. The enterence and end regions were 20 cm long, based on previous work (Brenner 1961, Tanner 1963, Yu *et al.* 1979) resulting in a measuring section length of 60 cm. Unfortunately, curvature was observed in the lower sections of the triangular and square cylinder, and hence the measuring sections of these cylinders were reduced from 20 to 40 cm from the top.

---

†Present address: Central Research Laboratories, Ebara-Infilco Co., Fujisawa, Kanagawa, Japan 251.

‡Present address: Design Group, Form Division, Manufacturing Machine Department, Canon Co. 3-20-1, Shimomaruko, Ohta-ku, Tokyo, Japan 146.

§Author for correspondence.

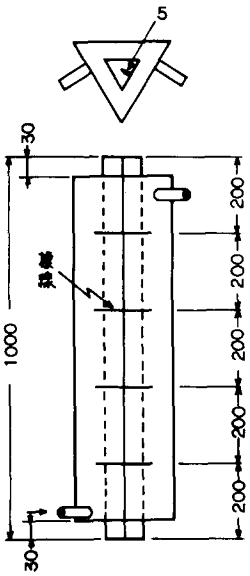

Figure 1. Triangular cylinder.

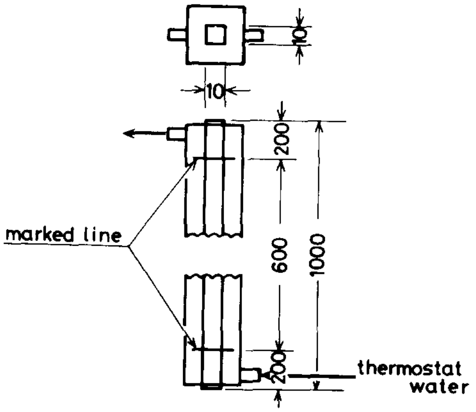

Figure 2. Square cylinder.

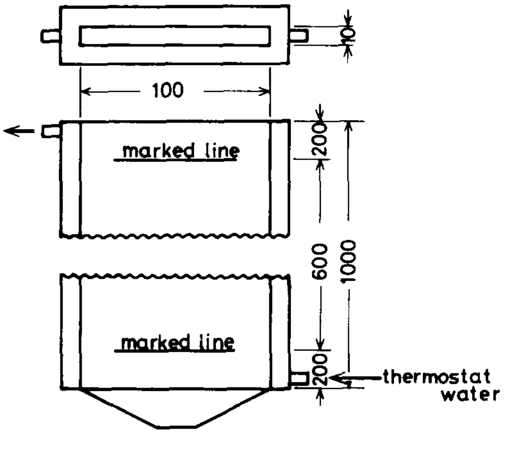

Figure 3. Parallel plates.

The apparatus were maintained precisely vertically by making it accurately parallel to a thread suspending a weight. The apparatus were filled with the aqueous millet jelley solution, which was highly viscous, but still a Newtonian fluid (Nakagawa 1975). For each run the aqueous millet jelley solutions in the apparatus were maintained at constant temperatures of 25 and 30°C by flowing water in thermostat into the jacket. The viscosities of the aqueous millet jelley solutions were measured at 20, 25, 30, 35 and 40°C by the Hoeppler falling ball viscometer, and were 1805 and 809.0 centipoise at 25 and 30°C for the experiments with the triangular cylinder, respectively and were 4420 and 2490 centipoise at 25 and 30°C for the experiments with the square cylinder and parallel plates. The densities of the aqueous millet jelley solutions were measured at 20, 25, 30, 35 and 40°C by specific gravity bottle in a thermostat.

The single ball bearings of diameters of 1.00, 2.00, 3.00, 4.00, 5.00, 6.00, 7.00, 8.00 and 9.00 mm were dropped into the centers of the tops of the apparatus. The times for the single ball bearings to pass through the measuring sections of the apparatus were measured by a stopwatch. The experiments were repeated three times for each run. The ball bearings were conditioned in the weight measuring bottle filled with the same aqueous millet jelley solutions in the thermostat and at the same temperature as the experimental apparatus.

3. EXPERIMENTAL RESULTS AND DISCUSSIONS

The single ball bearings were observed to settle along the center line in the measuring sections of the apparatus with no rotation. In the case that the ball bearing was dropped along the inside wall at the top of the apparatus, the ball bearing approached the center line with rotation and settled with no rotation along the centerline of the apparatus within the entrance region. These phenomena are naturally reasonable, as the viscous dissipation, namely entropy production rate becomes the minimum in the case that the solid sphere settles along the centerline of the apparatus. The steady state settling velocities, u_w were calculated from the times for which the single ball bearings passed through the measuring sections of the apparatus.

In the creeping flow region the following relationship holds between the free settling velocity, u_F and the settling velocity influenced by the wall, u_w.

$$F(r) = \frac{u_F}{u_w}. \qquad [1]$$

The free settling velocities were calculated from Stokes law, and the Reynolds numbers, of which representative velocities are free and hindered settling velocities, u_F and u_w, ranged from 0.000241 to 0.699 and from 0.000216 to 0.0395, respectively.

Figures 4 and 5 show semi-log plots of the wall correction factors of single solid spheres in triangular and square cylinders vs the dimensionless radii of the solid spheres, respectively.

Figure 6 shows a plot of the wall correction factors of single solid spheres in parallel plates vs the dimensionless radii of the solid spheres.

The clearance between the solid sphere of a fixed diameter and the wall of the apparatus increases with the order of the triangular and square cylinders, and parallel plates. The wall correction factors increases with decreasing clearance, as seen in figures 4–6. All the data negligibly scatter. In each run at least two data were measured and they negligibly scatter. Hence, one point of the data indicates at least two measurements in figures 4–6.

In the case of circular cylinders, the solid sphere of necessity settled with rotation within the very small clearances, even though precautions were taken. This rotation phenomena has been analysed by lubrication theory (Christopherson & Dowson 1959, Bungay & Brenner 1973). However, this rotation phenomena was rarely observed in this study, as the clearance between

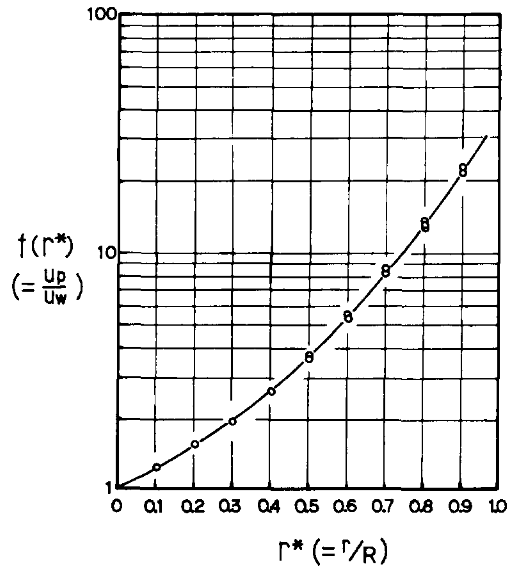

Figure 4. Experimental wall correction factors of single solid spheres in a triangular cylinder.

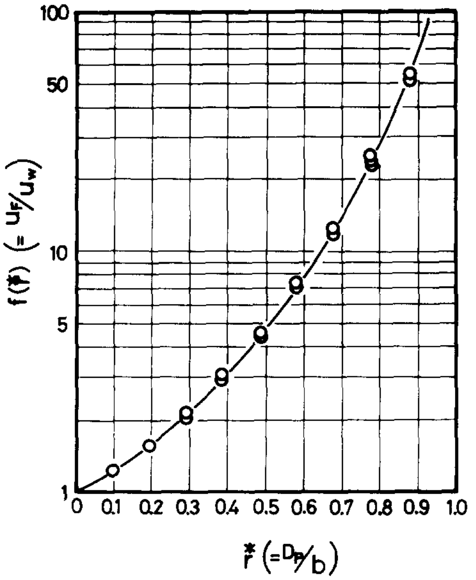

Figure 5. Wall correction factors of single solid spheres in a square cylinder.

the solid sphere of a fixed diameter and the inside wall of the triangular and a square cylinders and parallel plates is always larger than that between the solid sphere of the same diameter and the circular cylinder.

In the parallel plates the ball bearing sometimes settled with rotation along the centerline in the case of small clearances and these data were plotted by the triangular points in figure 6, but these phenomena were quite rare.

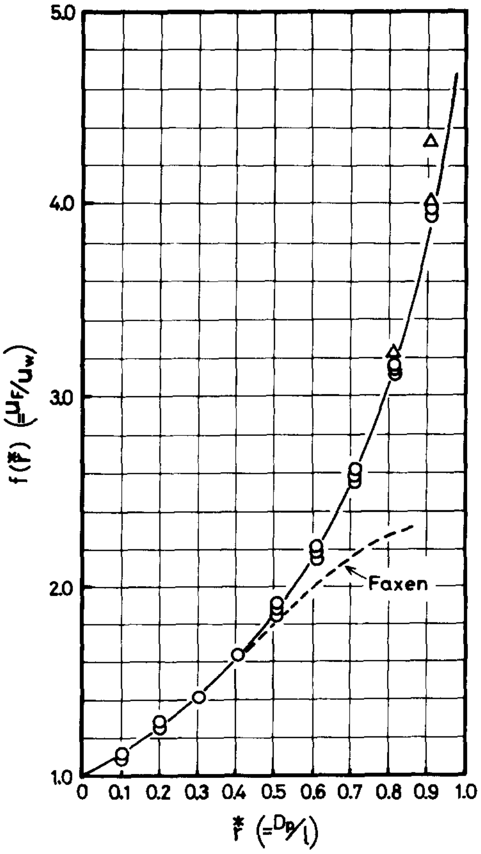

Figure 6. Wall correction factors of single solid spheres in parallel plates.

There is no study on the theoretical and experimental wall correction factors of the triangular and square cylinders, but Faxen (1922) obtained a theoretical wall correction factor for parallel plates by a series expansion solution, which is also plotted in Fig. 6. The series expansion solution by Faxen agreed very well with our experimental data up to $\overset{*}{r} = 0.4$, as it expanded a series in the vicinity of the center of $\overset{*}{r} = 0{,}0$. In this study the ratio of the diameter of the largest solid sphere to the longer width of the parallel plates is about 10 and hence the wall effect of the larger width is negligibly small.

The wall correction factors in the case of no rotation so obtained were correlated by the following 19th order equations by the least square method, the formulae of which are simular to the series expansion solution. The coefficients of the 19th order equations were tabulated table 1.

$$F(\overset{*}{r}) = \frac{1}{R_0 + R_1,\ \overset{*}{r} + R_2,\ \overset{*}{r}^2 + \cdots + R_{17},\ \overset{*}{r}^{17} + R_{18},\ \overset{*}{r}^{18} + R_{19}\ \overset{*}{r}^{19}} \qquad [2]$$

The standard variance is expressed by the following equation.

$$S = \sqrt{\left(\frac{1}{N}\sum_{i=1}^{N}[Fi(\overset{*}{r}) - F(\overset{*}{r})]^2\right)}.$$

The standard variances were 0.261, 0.485 and 0.0185 for the triangular and square cylinders and parallel plates, respectively and all the standard variances were considerably small, compared to the average wall correction factors, $F(r)$.

The triangular and square cylinders and parallel plates can be used as a falling ball viscometer from the wall correction factors so obtained. The viscometers of the triangular and square cylinders, and parallel plates can be easily made from the flat-transparent plates, which is more convienient than the conventional falling ball viscometer of the circular cylinder, of which the cross-section must be precisely circular. Furthermore, in the very small clearances

Table 1. Coefficients of polynomials by the least square method

	triagnlar cylinder		square cylinder		parallel plates	
Ro	1.0000000		1.0000000		1.0000000	10^{0}
R_{1}	-0.1524694	10^{1}	-0.1923777	10^{1}	-0.4027060	10^{0}
R_{2}	-0.9356945	10^{1}	0.1649393	10^{1}	-0.8435362	10^{1}
R_{3}	0.6788950	10^{2}	-0.1153624	10^{2}	0.3487996	10^{2}
R_{4}	-0.1634936	10^{3}	0.2682020	10^{2}	-0.2359584	10^{2}
R_{5}	0.6563649	10^{2}	0.1367386	10^{2}	-0.1193919	10^{3}
R_{6}	0.1929998	10^{3}	-0.5060226	10^{2}	0.1362242	10^{3}
R_{7}	0.4729873	10^{2}	-0.1042480	10^{3}	0.1601959	10^{3}
R_{8}	-0.3751033	10^{3}	0.1170802	10^{3}	-0.4106427	10^{1}
R_{9}	0.2752887	10^{3}	0.2395431	10^{3}	-0.3171554	10^{3}
R_{10}	-0.1190656	10^{4}	-0.1757552	10^{3}	-0.1989548	10^{3}
R_{11}	0.6542166	10^{3}	0.1097079	10^{2}	0.4608181	10^{2}
R_{12}	0.2075038	10^{4}	-0.2409061	10^{3}	0.3581750	10^{3}
R_{13}	0.3268518	10^{3}	-0.9802373	10^{2}	0.2128604	10^{3}
R_{14}	-0.2014247	10^{4}	0.1344775	10^{3}	0.2338137	10^{3}
R_{15}	-0.2869939	10^{4}	0.2037087	10^{3}	-0.8912624	10^{2}
R_{16}	0.1553641	10^{4}	0.9298401	10^{2}	-0.6472198	10^{2}
R_{17}	0.1483049	10^{4}	-0.1153909	10^{3}	-0.2528621	10^{3}
R_{18}	0.2078562	10^{4}	0.2791606	10^{3}	0.2000882	10^{3}
R_{19}	-0.2211987	10^{4}	-0.3434466	10^{3}	0.2897953	10^{3}

the rotation phenomena rarely occurs in the triangular and square cylinders, and parallel plates, but always occur in the circular cylinder, which causes an inaccurate measurement of viscosity.

4. CONCLUSION

The wall correction factors of single solid spheres were experimentally measured in triangular and square cylinders, and parallel plates in the creeping flow region.

In the future the wall correction factors in such apparatus should be theoretically studied by series expansions and numerical methods and be compared with these experimental data.

NOMENCLATURE

a	the length of the perpendicular line from the center to a side of the triangular cylinder, half of the length of a side of the square cylinder or half of the shorter width of the paralled plates, cm
$F(\overset{*}{r})$	wall correction factor
$Fi(\overset{*}{r})$	measured wall correction factor
N	number of the data of the measured wall correction factors
r	radius of a solid sphere, cm
$\overset{*}{r}$	dimensionless radius of a solid sphere = r/a
R_0–R_{19}	coefficients of the 19th order equation
S	standard variance
u_f	free settling velocity of a single soid sphere calculated from Stokes law, cm/s
u_w	settling velocity of a single solid sphere influenced by wall, cm/s

REFERENCES

Brenner, H. 1961 The slow motion of a sphere through a viscous fluid towards a plane surface. *Chem. Engng Sci.* **16**, 242–251.

Bungay, P. M. & Brenner, H. 1973 The motion of a closely-fitting sphere in a fluid-filled tube. *Int. J. Multiphase Flow* **1**, 25–56.

Christopherson, D. G. & Dowson, D. 1959 An example of minimum energy dissipation in viscous flow. *Proc. R. Soc.* **A251**, 550–564.

Faxen, H. 1922 Der Widerstand gagen die Bewegung einer starren Kugel in einer zaehen Fluessigkeit, die zwischen zwei parallelen ebenen Waenden eingeschlossen ist. *Ann. Phys.* **68**, 89–119.

Haberman, W. 1956 Wall effect for rigid and fluid spheres in slow motion. Ph. D. Thesis, University of Maryland.

Ishii, T. 1965 Behavior of liquid-solid fluidization. *Kagaku Kogaku* **29**, 915–921.

Ito, S. 1957 *Kagaku-Kogaku-Nyumon.*

Ito, S. 1962 Fluid mechanics in packed beds, fluidization and filtration. *Kagaku Kogaku* **26**, 1109–1115.

Iwaoka, M. & Ishii, T. 1979 Experimental wall correction factors of single solid spheres in circular cylinders. *J. Chem. Engng Japan* **12**, 239–242.

Nakagawa, T. 1975 Rheology, current topics. *Math. Sci.* **140**, 16–21.

Paine, P. L. & Scherr, P. 1975 Drag coefficients for the movement of rigid spheres through liquid-filled cylindrical pores. *Biophys. J.* **15**, 1087–1091.

Tanner, R. I. 1963 End effects in falling-ball viscometry. *J. Fluid. Mech.* **17**, 161–170.

Yu, S., Fukui, Y., Miyatani, Y., Murakami, Y. & Tohtani, G. 1979 Measurement of drag force of a sphere towards a plane surface. *Proc. Ann. Meeting of Soc. Chem. Engrs Japan*, F120, pp. 503–504.